Proceedings of the 11th World Congress in Mechanism and Machine Science
April 1-4, 2004, Tianjin, China. -Vol.3, Mechatronics, - P. 1331-1335

On Dynamics of Mechatronic Systems with Incomplete Differential Programs of Motion

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Abstract: Formalism is created describing the dynamics of mechatronic systems with incomplete differential programs of motion. The motions of such systems are featured as a motion of its constrained Affix in R_S-space. The theorem of structure of r-derivative of the Affix constraint force is proved. On its basis the Affix motion equations and series of their covariant forms are derived. The Extension of Maxwell – LaGrange theory for electromechanical systems with arbitrary incomplete differential programs of motion is made. Examples are given.

Keywords: Mechatronic, Incomplete differential programs of motion, Arbitrary nonholonomic constraints, Affix equations of motion, Covariant forms of motion's equations, Equations of electromechanical systems with the arbitrary incomplete differential programs of motion.

1 Introduction

Mechatronic's development leads to necessity of elaboration of adequate mechanical models of motion of systems with *incomplete differential programs of motion* [1]. It is known, than the large class of system's motions, controlled on the program, can be described as a class of motions with arbitrary holonomic and nonholonomic constraints [1-13]. In the frames of a theory it can be determined as differential constraints with any orders, which set a program of motion. In mechatronics it is accepted to term bilateral constraints as controllable constraints, and constraints equations - representations of the programs of motions (program of motion) [1]. Program of motion is called *complete* if it determines law of a controlled object. Otherwise it is called *incomplete*. It is known that the problem of motion of systems with program of motion is mainly decided in the classic mechanics [1, 12]. If differential motion's programs of any orders, the problem can be decided in the frames of classic mechanics spreaded on such system's motion. Results of our version of the Extension of mechanics are given, for instance in the articles [7-11].

In this manuscript the dynamic's elements of mechatronic system with incomplete differential programs of motion without taking into account control errors are presented. The programs describe so-called open-looped control systems [1,1 2].

Analytical covariant forms of motions equations of such systems were obtained on the base of an approach, connected with introduction of an Affix for constrained material system [7, 13-15]. The Affix moves in the E_{3N} on the diversity of R_s, constrained by differential ties – programs of motion. The next analytical forms of motion's equations are discussed: "in generalized forces" – R_sQ, "Appell-like" – R_sA, "LaGrange-like" – R_sL, "LaGrange-Maxwell-like" – R_sL-M. All of these forms are united by the form of motion's equation in "generalized force factors" – $RQ^{(r)}$-form, introduced in the report.

Let's determine the systems, motion of which is constrained by tie's equations, beginning from nonlinear in relation with acceleration as systems with incomplete differential programs of motion of top orders [7,13,15]. As examples of problems of such system's motion the problems are adduced about:

a) orbital or angular maneuver of space vehicles, for example, with guidance module by acceleration of center of mass or angular acceleration;
b) "super weak" docking of space vehicles;
c) dynamics of testing machines controlled by module of acceleration or jerk;
d) "super weak" motions of technological robots and manipulators transferring, for example, detonating load;
e) dynamics of self-acting controllable nonstop drive units used, for example, in aviation and rocket systems.

To realize programmable motion of the objects it is necessary to have a drive, which could make a program of motion. To develop such drive it is necessary to have both adequate models of motion of control objects and adequate automatic control algorithms. Let's note, that the lack of the right equations of motion of nonholonomic high order systems at the present does certain principles and results of the report [16] to be wrong. Let's mark also that the problem of algorithm's synthesis for control by high order derivatives have been decided in main by the group of scientists of prof. Vostrikov A.S. and is discussed in [17] for example.

2 Choice of problem

It is known there are two sets of problem of investigation of motion controllable on the program historically. According with first of them material system under active control forces moves such mode that the equations of constraints are satisfied. In the case the main goal is receiving of a set of equation of motion which has not unknown constraints reaction [13-15].

According with the second set of problem both system's motion and adding forces-reactions of constraints together with control forces provide the motion in according with constraints conditions [1,2,10-12,15]. In the case the problem is mixed dynamic problem. It is solved by mode of classic manuscript [18] in holonomic coordinates on the base of LaGrange theory. When the most of problems of applied mechanics and machine theory are solving the method of solution of the problem gives real interest because of in its frames motion equations are developed and analyzed by convenient for engineers language of force factors, i.e. of forces and torques. In conjunction with it the method is considered in the report only.

3 The basic theorem of theory of systems with nonholonomic constraints of any orders

The Extension of Classic Mechanics for nonholonomic systems of any orders-systems with incomplete differential programs of motion – is strictly connected with the theorem of structure of an *r*-derivative of a *Force Factor* of Affix constraints [11], which determines both its "smoothing degree" and Affix motion equations of considering systems.

The theorem:

Let's describe the motion of constrained material system of the mass *M* as motion of its Affix in E_{3N} - space on diversity R_s [7-11]. It is limited by holonomic and nonholonomic constraints φ^p. The equations of motion are defined by the theorem of structure of an *r*-derivative of an Affix constraint reaction :

Let the motion of system is in correspondence with

$$\begin{cases} f^p(t,q^j,\dot{q}^j,\ldots,\overset{(k)}{q^j}) = \varphi^p(t,x_i,\dot{x}_i,\ldots,\overset{(k)}{x_i}) = 0, \\ j = 1,2,\ldots,s;\ldots i = 1,2,\ldots 3N;\ldots p < s \end{cases} \quad (1)$$

where $\boldsymbol{x} = \boldsymbol{x}(t,x_i) = \boldsymbol{x}(t,q^j)$ -Affix's radius-vector identified in a Euclidean space E_{3N} selects in E_{3N} a Riemannian variety R_S ; $x_i = \chi_i\sqrt{\mu_i}, \mu_i = m_i / M$, q^j - system's generalized coordinates.

Then $$\overset{(r)}{\boldsymbol{R}} = \lambda_p \partial_{\boldsymbol{k}} \varphi^p + \boldsymbol{T}_k(t,x_i,\ldots\overset{(r+1)}{x_i}), \quad (2)$$

and the closed set of equations of constrained motion of Affix in E_{3N} takes the form

$$\begin{cases} M\overset{(r+2)}{\boldsymbol{x}} \overset{(r)}{=} \boldsymbol{F} + \lambda_p \partial_k \varphi^p + \boldsymbol{T}_k ; \\ d_t \varphi^p = \overset{(r+2)}{\boldsymbol{x}} \partial_k \varphi^p + \Psi^p(t,x_i,\ldots\overset{(r+1)}{x_i}) = 0; \\ \partial_k = \partial / \partial \overset{(k)}{\boldsymbol{x}}, d_t = d/dt, \\ if \;\; k \le 1 \;\; then \;\; r = 0, \;\; if \;\; k \ge 2 \;\; then \;\; r = k - q. \end{cases} \quad (3)$$

Here identified vectors of *the force factors* of active forces are $\boldsymbol{F}(F_i)$, $F_i = f_i/\sqrt{\mu_i}$, f_i is *i*-component of active force. On a double index the summing is produced; *q*=1 for nonlinear, and *q*=2 for linear on $\overset{(k)}{x_i}$ constraints; an external vector-function $\boldsymbol{T}_k \perp \partial_k \varphi^p$. The LaGrange's factors $\lambda_p = \lambda_p(t) = \overset{(r+2)}{\mu_p} \overset{(r)}{=} \eta_p$ represented in the form adapted for numerical solution may be found as a result of solving of the set of equations.

For linear on $\overset{(k)}{x_i}$ constraints

$\varphi^p = A_i^p(t,x_i,\ldots\overset{(k-1)}{x_i})\overset{(k)}{x_i} + b^p(t,x_i,\ldots\overset{(k-1)}{x_i})$ function Ψ^p look like $\Psi^p = b^p(t,x_i,\ldots\overset{(k-1)}{x_i})$. For other constraints

$\psi^p = \partial_t \varphi^p + \partial_0 \varphi^p \dot{x}_i + \ldots + \partial_{(k-1)} \varphi^p \overset{(k)}{x_i}$.

Uncomplicated proof of the theorem is not given here. Analysis of expression (2) shows that for realization of incomplete program of motion it is enough to determine function $\overset{(r)}{\boldsymbol{R}}$ with number of *k*. Naturally the control of incomplete program of motion may be not optimal. "Reference part of control" in the frames of the motion's program is determined by form of function $\boldsymbol{T}_k$ only. Let's note that an introducing in the system the part of control according to function $\boldsymbol{T}_k$ leads to different modes of system's motion when forms of the function $\boldsymbol{T}_k$ are different. Also note that k-gradient control is according with so called "ideal" constraints (following Gartung-Dobronravov) [13].

4 Covariant forms of motion's equations of the systems with differential constraints

Obviously that equation (3) is really unsolvable if the system has even a few of particles. Within the framework of scientific traditions there is a problem on passage from a set of equations (3) to equivalent to her analytical covariant set of equations in generalized coordinates. Actually it means "a passage from a mechanics of particles (bodies) to a mechanics of motions of interacting partial subsystems, forming the system"[12]. A number of forms of motion's equations are possible here. However forms R_sQ, R_sA, R_sL and intended for electromechanical and mechatronic systems R_sL-*M*-form are more traditional.

To infer analytical forms of motion's equations the set of equation (3) was used. In the set *k*-gradient part is determined by the method of the LaGrange's factors. Possibility of calculation of generalized inertia forces and other physical values by algorithms of calculation of those values *for holonomic systems* takes place only if that mode of solution in nonholonomic problem is used. Here nonholonomic motion is described in terms of holonomic motion: the vectors $\boldsymbol{F}, \boldsymbol{\Phi}$ can be submitted according to [7,15] as

$$\boldsymbol{F} = Q_i^F \boldsymbol{e}^i, \;\; \boldsymbol{\Phi} = -M(\ddot{\boldsymbol{x}})_i \boldsymbol{e}^i = Q_i^{\Phi} \boldsymbol{e}^i. \quad (4)$$

Here: Q_i^F is generalized force; Q_i^{Φ} is generalized inertia force; $\boldsymbol{e}^i$ - Affix-coordinate vectors of mutual base in tangential to R_S space E_{3N} [7-9],

$$\mathrm{e}^i = g^{ij}\mathrm{e}_j; \;\; \mathrm{e}_j = \partial_0 \mathrm{x} = \partial_1 \dot{\mathrm{x}} = \ldots \quad (5)$$

g^{ij} are contravariant components of the metrical tensor of the configurations space R_s, determined by expression for kinetic energy of the system [14].

Let's note that Q_i^{Φ} will be calculated by LaGrange's, Nielsen's, Appell's and so on procedures [7, 13]:

$$Q_i^{\Phi} = -(d_t(\partial T / \partial \dot{q}^i) - \partial T / \partial q^i) = \\ = -(\partial \dot{T} / \partial \dot{q}^i - 2\partial T / \partial q^i) = -\partial S / \partial \ddot{q}^i = ... \quad (6)$$

T – kinetic energy of the system; S – Appell's function [14].

Analytical forms of motion's equations noted above are obtained from a system (3) as outcome it of scalar multiplication on vectors $\boldsymbol{e}_j$ and series of simple mathematical transformations. They accept a following aspect:

R_sQ – form

$$\begin{cases} d_t^{(r)}((Q_i^{\Phi} + Q_i^F)\mathbf{e}^i)\mathbf{e}_j + \lambda_p \partial_k^j f^p + Q_{j(T)}^{(r)} = 0 \\ \overset{(r+2)}{q^j} \partial_k^j f^p + W^p(t, q^j, ... \overset{(r+1)}{q^j}) = 0 \\ i, j = 1,2..., r+2 \quad p \le r+1, \quad q = 1,2, \\ if \;\; k \le 1 \;\; then \;\; r = 0, \;\; if \;\; k \ge 2 \;\; then \;\; r = k - q. \end{cases} \quad (7)$$

Here $d_t^{(r)} = d^{(r)} / dt^{(r)}, \partial_k^j = \partial / \partial \overset{(k)}{q^j}$.

R_sA – form

$$\begin{cases} \partial_s^j K_{[s]} = \lambda_p \partial_k^j f^p + Q_{j(F)}^{(r)} + Q_{j(T)}^{(r)} = 0 \\ \overset{(s)}{q^j} \partial_k^j f^p + W^p(t, q^j, ... \overset{(s-1)}{q^j}) = 0 \\ j = 1,2..., s, \;\; p \le s-1, \;\; s = r+2, \;\; q = 1,2, \\ if \;\; k \le 1 \;\; then \;\; r = 0, \;\; if \;\; k \ge 2 \;\; then \;\; r = k - q. \end{cases} \quad (8)$$

Here according [7-11] K_s is *the universal dynamic measure of motion – the Kineta:*

$$K_s = M(\overset{(s)}{\mathbf{x}})^2 / 2 = \frac{1}{2} \sum_{\ell=1}^{s} m_\ell (\overset{(s)}{\mathbf{r}_\ell})^2 . \quad (9)$$

$K_{[s]}$ is the kineta's part, quadratically depending on $\overset{(s)}{\boldsymbol{q}^j}$. Let's mark, that $K_1 = T$ and $K_2 = S$. For kineta is valid the theorem such as the theorem of Konig [8]:

$$K_s = K_s^{(c)} + K_s^{(i/c)} \quad (10)$$

$Q_{j(\ )}^{(r)}$ is *generalized force factor of the r –order* [11]:

$$Q_{j(F)}^{(r)} = (\overset{(r)}{\boldsymbol{F}} \cdot \boldsymbol{e}_j), \quad Q_{j(T)}^{(r)} = (\boldsymbol{T}_k \cdot \boldsymbol{e}_j) \quad (11)$$

Let's mark, that if $Q_{j(F)}^{(0)} = Q_j^F$, and at *r=0* the motion's equations in the R_sQ –form became customary equations of motion in generalized forces. The motion's equations in the "R_sA"- form turn to the Appell's equations [13-15].

Let's consider R_sL-form of motion's equations. These are reduced as forms represented above and looks like:

$$\begin{cases} \hat{\Lambda}_j^{(r+1)}(K_{(r+1)}) = \lambda_p \partial_k^j f^p + Q_{j(F)}^{(r)} + Q_{j(T)}^{(r)} \\ \overset{(r+2)}{q^j} \partial_k^j f^p + W^p(t, q^j, ... \overset{(r+1)}{q^j}) = 0 \\ j = 1,2..., r+2, \;\; p \le r+1, \;\; q = 1,2, \\ if \;\; k \le 1 \;\; then \;\; r = 0, \;\; if \;\; k \ge 2 \;\; then \;\; r = k - q. \end{cases} \quad (12)$$

Let's term $\hat{\Lambda}_j^{(r+1)} = d_t \partial_{r+1}^j - (r+1)^{-1} \partial_r^j$ as the Euler – LaGrange operator of the order (r+1) [7].

Let's note that if *r*=0 equation (12) becomes LaGrange equations.

6 Equations of motion in generalized force factor

Let's introduce *the generalized force factor of inertia* as

$$Q_{j(\Phi)}^{(r)} = -\partial_s^j K_{[s]} = -\hat{\Lambda}_j^{(r+1)}(K_{(r+1)}), \quad (13) \\ j = 1,2,..., r+2, \quad r = k - q, \; q = 1,2.$$

and also determine $Q_{\lambda j}^{(r)}$ as

$$Q_{j(\lambda)}^{(r)} = \lambda_p \partial_k^j f^p , \quad (14)$$

Then motion's equations in "*generalized force factors*" ($RQ^{(r)}$-form equations) have compact integrated forms, received above:

$$\begin{cases} Q_{j(\Phi)}^{(r)} + Q_{j(F)}^{(r)} + Q_{j(\lambda)}^{(r)} + Q_{j(T)}^{(r)} = 0 \\ \overset{(r+2)}{q^j} \partial_k^j f^p + W^p(t, q^j, ... \overset{(r+1)}{q^j}) = 0 \\ j = 1,2..., r+2, \quad p \le r+1, \quad q = 1,2, \\ if \;\; k \le 1 \;\; then \;\; r = 0, \;\; if \;\; k \ge 2 \;\; then \;\; r = k - q \end{cases} \quad (15)$$

The form of motion's equations as well as R_sQ-form accepts the introduction of *Representation of Interacting Partial Motions and Interacting Bodies.* These are subscribed in details in our works [10, 12]. Because of it the question will not discussed here.

7 Equations of motion of electromechanical systems with differential constraints

The electromechanical analogy similar 1st electromechanical analogy of classic mechanics takes place in dynamics of systems with differential constraints as (1).There are both mechanical and electrical generalized coordinates here. It is based on the Extension of Maxwell's postulate [11]. The postulate is formulated as:

Motion's equations of controlled electromechanical systems with incomplete differential programs (as (1)) of motions are formed in LaGrange's - R_sL or Appell's - R_sA forms.

For such systems R_sL-*M*-form of the equations are as follows:

$$\begin{cases} \overset{\wedge}{\Lambda}{}^{(r+1)}_j (L_{(r+1)}) = -\partial^j_{r+1} R_{(r+1)} + Q^{(r)}_{j(F)} + Q^{(r)}_{j(\lambda)} + Q^{(r)}_{j(T)} \\ \overset{(s)}{q^j}\, \partial^j_k f^p + W^p(t, q^j, \dots \overset{(s-1)}{q^j}) = 0 \\ j = 1,2\dots, s,\quad p \le s-1,\quad s = r+2,\quad q = 1,2, \\ if\ \ k \le 1\ then\ r = 0,\ if\ \ k \ge 2\ then\ r = k-q. \end{cases} \tag{16}$$

Here $L_{(r+1)}$ and $R_{(r+1)}$ – LaGrange's and Raleigh's functions. There are calculated as:

$$L_{(r+1)} = L^{mech}_{(r+1)} + L^{elec}_{(r+1)}, \quad R_{(r+1)} = R^{mech}_{(r+1)} + R^{elec}_{(r+1)}, \tag{17}$$

$$L^{mech}_{(r+1)} = K^{mech}_{(r+1)} - \Pi^{mech}_{(r)}, \quad L^{elec}_{(r+1)} = K^{elec}_{(r+1)} - \Pi^{elec}_{(r)}. \tag{18}$$

$\Pi_{(r)}$ is the analog of member r of potential energy Π in such electromechanical systems for which the energy may be calculated as

$$\Pi = c_{ij} q_i q_i / 2\,. \tag{19}$$

Then the function $\Pi_{(r)}$ and $R_{(r+1)}$ may be written in form

$$\Pi_{(r)} = c_{ij} \overset{(r)}{q_i} \overset{(r)}{q_i} / 2, \qquad R_{(r+1)} = r_{ij} \overset{(r+1)}{q_i} \overset{(r+1)}{q_j} / 2\,, \tag{20}$$

where C_{ij} and r_{ij} are the quasi-elastic and dissipative coefficients of electromechanical system.

If the potential energy of the system is represented in more general form, the equations of motion have Appell-like form (R_sA-form):

$$\begin{cases} \partial^j_s K_{[s]} = \lambda_p \partial^j_k f^p - \partial^j_{r+1} R_{(r+1)} + Q^{(r)}_{j(\Pi)} + Q^{(r)}_{j(F)} + Q^{(r)}_{j(T)} \\ \overset{(s)}{q^j}\, \partial^j_k f^p + W^p(t, q^j, \dots \overset{(s-1)}{q^j}) = 0 \\ j = 1,2\dots, s,\quad p \le s-1,\quad s = r+2,\quad q = 1,2, \\ if\ \ k \le 1\ then\ \ r = 0,\ if\ \ k \ge 2\ then\ \ r = k-q. \end{cases} \tag{21}$$

Here

$$Q^{(r)}_{j(\Pi)} = (d^{(r)}_t(-\partial_0 \Pi) \cdot \boldsymbol{e}_j) \tag{22}$$

Equations (16) and (21) adequately describe the motion of electromechanical and mechatronic systems with arbitrary complete and incomplete differential programs of motion as (1). If in the equation (1) there are mechanical or electrical $q^j, \dot{q}^j \dots$ only then $Q^{(r)}_{j(\lambda)}$ is obtained from the problem's conditions. The example of the problem will be presented below.

8 Examples

Let's complete motion's equations of open – looped control model of testing unit with three degrees of freedom for dynamic tests of the equipment in terms of quasi-velocities and quasi-accelerations. Motion of the unit goes in according with incomplete differential program. Let $\boldsymbol{\varepsilon}$ -gradient control exists. It is referenced by incomplete motion's program:

$$f = (\boldsymbol{\varepsilon} \cdot \boldsymbol{\varepsilon}) - \boldsymbol{\varepsilon}^2_0(t) = 0\,, \tag{23}$$

Here $\boldsymbol{\varepsilon}$ - angle acceleration of the unit with an object of testing.

Dynamic motion's equations are forming in R_SA-form

$$\begin{cases} \partial K_3 / \partial \dot{\varepsilon}^j = \eta \partial f / \partial \dot{\varepsilon}^j\,, \quad j = x,y,z, \\ d_t f = 0. \end{cases} \tag{24}$$

The set of dynamic equations of program motion in moving axis looks like

$$\begin{cases} I_x \dot{\varepsilon}_x - \omega^2 I_x \omega_x + I_2(\omega_y \varepsilon_z + 2\varepsilon_y \omega_z) - \\ - I_3(\omega_z \varepsilon_y + 2\varepsilon_z \omega_y) - 2\dot{\eta}\,\varepsilon_x = 0 \\ I_y \dot{\varepsilon}_y - \omega^2 I_y \omega_y + I_3(\omega_z \varepsilon_x + 2\varepsilon_z \omega_x) - \\ - I_1(\omega_x \varepsilon_z + 2\varepsilon_x \omega_z) - 2\dot{\eta}\,\varepsilon_x = 0 \\ I_z \dot{\varepsilon}_z - \omega^2 I_z \omega_z + I_1(\omega_x \varepsilon_y + 2\omega_y \varepsilon_x) - \\ - I_2(\omega_y \varepsilon_x + 2\varepsilon_y \omega_x) - 2\dot{\eta}\,\varepsilon_z = 0 \\ (\varepsilon_x \cdot \dot{\varepsilon}_x + \varepsilon_y \cdot \dot{\varepsilon}_y + \varepsilon_z \cdot \dot{\varepsilon}_z) - \varepsilon_0 \cdot \dot{\varepsilon}_0(t) = 0 \\ \varepsilon_x = \dot{\omega}_x, \varepsilon_y = \dot{\omega}_y, \ \ \varepsilon_y = \dot{\omega}_y, \ \varepsilon_z = \dot{\omega}_z. \end{cases} \tag{25}$$

Here

$$I_1 = (-I_x + I_y + I_z)/2; \quad I_2 = (I_x - I_y - I_z)/2;$$
$$I_3 = (I_x + I_y + I_z)/2.$$

It means that the set of equations is added for example by kinematical Euler's equations.

Let's make the set of equations of the plane model (z,φ) of electromechanical unit. Its schematic diagram is presented on the Figure 1.

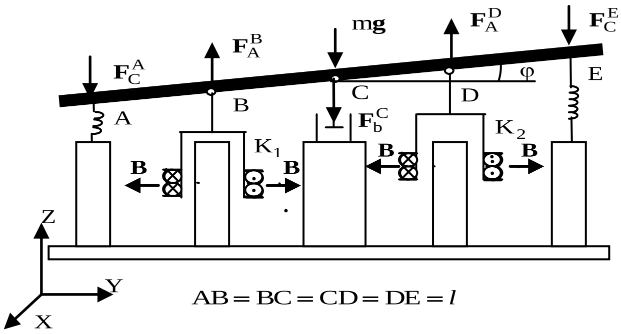

Figure1

Here

m-mass of system, $J_{cx} = J_c$ - moment of inertia of system with respect to an axis x;

c-stiffness of springs, b-damping coefficient of system;

$\boldsymbol{B}$-magnetic field inductance in the air gap;

R-active resistance of the coil, L-inductance of the coil;

n-a number of wiring of the coil, r-middle radius of the coil.

Control voltages U_1, U_2 feed the coils. Ampere's forces $\boldsymbol{F}^B_A, \boldsymbol{F}^D_A$ act as control forces.

The next values we shell count as mechanical generalized coordinates of the system:

a) displacement of point C from static equilibrium state $q_{1mech} = z_c = x;$

b) angle φ. Let the angle φ will be restricted by little values.

Changes in the coils K_1 and K_2 $q_{1el} = q$, $q_{2el} = e$ are accepted as electrical generalized coordinates of the system. $I_q = \dot{q}$, $I_e = \dot{q}_e$

A motion control of the table of the testing unit is realized in according with incomplete differential programs of motion for example

$$f = K_2^{mech} - K_{20}^{mech} = S - S_o = 0 \qquad (26)$$

Let's get a set of motion's equations of electromechanical system in the R_sL-M-form:

$$\begin{cases} \hat{\Lambda}_x^{(2)} L_{(2)} = -\partial_2^x R_{(2)} + \dot{\eta}\,\partial f / \partial \ddot{x} \\ \hat{\Lambda}_\varphi^{(2)} L_{(2)} = -\partial_2^\varphi R_{(2)} + \dot{\eta}\,\partial f / \partial \ddot{\varphi} \\ \hat{\Lambda}_q^{(2)} L_{(2)} = -\partial_2^q R_{(2)} + \dot{\eta}\,\partial f / \partial \ddot{q} + \dot{U}_q \\ \hat{\Lambda}_e^{(2)} L_{(2)} = -\partial_2^e R_{(2)} + \dot{\eta}\,\partial f / \partial \ddot{e} + \dot{U}_e \\ d_t f = 0 \end{cases} \qquad (27)$$

Here $Q_{j(T)}^{(1)} = 0$

Let's note that Ampere's forces are

$$F_A^B = 2\pi rnBI_q, \; F_A^D = 2\pi rnBI_e \qquad (28)$$

Therefore the control generalized force factors $Q_{j(\lambda)}^{(1)}$ are

$$\begin{aligned} Q_{x(\lambda)}^{(1)} &= \dot{\eta}\,\partial f / \partial \ddot{x} = \dot{F}_A^B + \dot{F}_A^D = 2\pi rnB(\dot{I}_q + \dot{I}_e), \\ Q_{\varphi(\lambda)}^{(1)} &= \dot{\eta}\,\partial f / \partial \ddot{\varphi} = (\dot{F}_A^D - \dot{F}_A^B)l = 2\pi rnB(\dot{I}_{\mathrm{e}} - \dot{I}_q)l, \\ Q_{q(\lambda)}^{(1)} &= Q_{e(\lambda)}^{(1)} = 0. \end{aligned} \qquad (29)$$

Let's express from (29) $\dot{I}_q, \dot{I}_e$ and substitute it in (27). In result with it get the motion equations of electromechanical unit in respect with $x, \varphi, U_q, U_e, \eta$:

$$\begin{cases} m\dddot{x} + (b - \dot{\eta}m)\ddot{x} + c\dot{x} = 0 \\ J(\dddot{\varphi} - \dot{\eta}\ddot{\varphi} - \dot{\varphi}^3) + cl^2\dot{\varphi} = 0 \\ L\dfrac{d}{dt}(\dot{\eta}(ml\ddot{x} - J\ddot{\varphi})) + R\dot{\eta}(ml\ddot{x} - J\ddot{\varphi}) = 4\pi rnlB\dot{U}_q \\ L\dfrac{d}{dt}(\dot{\eta}(ml\ddot{x} + J\ddot{\varphi})) + R\dot{\eta}(ml\ddot{x} + J\ddot{\varphi}) = 4\pi rnlB\dot{U}_e \\ m\dddot{x}\ddot{x} + J(\dddot{\varphi} + 2(\dot{\varphi})^3)\ddot{\varphi} = 0 \end{cases} \qquad (29)$$

The equations of program motion with incomplete program for a testing unit of 6-degrees of freedom – imitator of machines moving [19] will be represented in our report. It doesn't show here because of large volume.

9 Conclusion

Received and presented materials are our contribution in the created theory of mechatronic systems with incomplete arbitrary differential programs of motion.

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