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ON INCREASING OF INTERNAL FRICTION WITH ELASTIC HYSTERESIS IN THE COMPRESSION MEMBER BARS OF VIBRATION PROTECTION SYSTEM

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Abstract
There are the results of theoretical researches, which explain the experimentally observed increasing of internal friction in the framework classical theory of elastic hysteresis inn the vibration protection systems, which use quasi-zero stiffness effect. Here elastic element was done as a bar, stiffness of which is decreasing by compressive force. The phenomenon is more marked when minimum stiffness bars takes place. It corresponds to the infra-low oscillation frequency in the system "bar – vibration protection object" (0,1-0,5 hertz)

Keywords: vibration protection system, bar, qvasi-zero stiffness, internal friction, infra-low frequency

1. Introduction

Increasing of the internal friction was displayed in the vibration protection systems, developed on the Department of Theoretical Mechanics & Strength of Material of Novosibirsk State Technical University under the guidance of professor Yuriev G.S The contents of experiments and some results are described in [1]. It is discovered that elastic hysteresis is essentially increased under condition of cycling strain of compression bar when longitudinal force P draw near approach to the critical force $P_{kr} = EJ(\pi/\ell)^2$. The regularity beside of cognizing interest has a practical importance. To realize necessary damping in the vibration protection systems with such elastic bars, for example.

During the last twenty years we have made techniques of synthesis of monolithic modules of qvasi-zero rigidity with given elastic, dissipative and geometrical characteristics. On their basis we created vibration protection systems for vibro-insulation of the sensitive equipments, elements and units of transport systems. They work in production of microelectronic goods and at the plants of the Ministry of Railways of Russia. The planed unique measuring system is a mechanical analogue of full-wave circuit Wheatstone (electric bridge Wheatstone), for diagnostics of latent defects of construction of flying devices, other vehicles and so on. Patents of Russia protect all products and techniques.

2. Theoretical results explaining the effect

Theoretical results explaining the effect are considered in the paper. This effect cannot be explained by possible influence of constructional friction since it is simply no in modules of qvasi-zero rigidity. The effect is also not explained by stress-strain state of a bar. The increasing of internal friction has explanation in the francs of classical theory of elastic hysteresis.

2.1. Statically model of effect

Let's consider the case of dead load of normal-compressive force P of a bar by vertical force F affixed in its center. As is known [2] is an expression for deflection of a bar without taking hysteresis losses was found by Timoshenko S.P. and has a form

$$y=(2F\ell^3/\pi^4 EJ)\sum_{m=1}^{\infty}(1/((2m+1)^4(1-\alpha/(2m+1)^2)sin((2m+1)\pi x/\ell)),\ \alpha=P/P_{kr}\to 1. \qquad (1)$$

Traditionally the calculation of internal losses is achieved with the average factor of absorption Ψ [3, 4]).

$$\Delta E=\Psi E \qquad (2)$$

Here ΔE - dispersion energy in the bar for one cycle of load; E - elastic strain energy.

These methods of the calculation of dispersion energy are absolutely competent when Ψ and E are not the functions of the parameter α. However, if Ψ and E, or at least one of these values is a function α, then the calculation of dispersion energy for one cycle of load can be done according to the laws of the mean.

$$\Delta E(\alpha)=\langle\Psi\rangle E(\alpha) \qquad (3)$$

Analysis of formula (3) proves (asseverate; claim) existence or absence of an anomalous increasing of losses (wastes) when $\alpha\to 1$.

Internal energy of a bar corresponding (1) is)

$$E=2F^2\ell^3/(\pi^4 EJ)\sum_{m=0}^{\infty}((2m+1)^2-\alpha)^{-2} \qquad (4)$$

Thus $$\Delta E\approx K(1-\alpha)^{-2}+K\sum_{m=1}^{\infty}(16m^2(2m+1)^{-1}) \qquad (5)$$

Here $K=2\langle\Psi\rangle F^2\ell^3/(\pi^4 EJ)$ and in the last sum the value $\alpha\to 1$substitutes for the unit, which has an influence on the result very little, as series is quickly-converging. Analysis of formula (5) achieves, that when $\alpha\to 1$ the increasing of hysteresis loss is observed.

For example internal friction for one load cycle $\Delta E(\alpha)$, when cycling static loading takes place increase as well as

$$\Delta E(\alpha)=\Delta E_0+\frac{2\langle\Psi\rangle F^2 l^3}{\pi^4 EJ}\cdot\frac{1}{(1-\alpha)^2} \qquad (6)$$

2.2. Dynamics model of effect

2.2.1.The effect under equivalent viscous friction

Let us consider a dynamic variant of solving problem**.** Reducing of internal losses to equivalent viscous friction $\sigma=E\varepsilon+r\dot{\varepsilon}$ the equation of transverse vibrations of the longitudinal- compression bar takes the form [3, 5].

$$EJ\frac{\partial^4 y}{\partial x^4}+rJ\frac{\partial^5 y}{\partial x^4\partial t}+P\frac{\partial^2 y}{\partial x^2}+\rho S\frac{\partial^2 y}{\partial t^2}=l^{-1}F(t)\delta(x-d) \qquad (7)$$

Let's look for the solution of equation (7) with the method of Timoshenko S.P. [2,5]. Let's present y as series by free forms of vibrations.

$$y = \sum_{m=1}^{\infty} y_m(t)\sin(\mathrm{m}\,\pi \mathrm{d/l}) \tag{8}$$

Than

$$y_m \exp(-\beta_m t)\sin(\omega_m t + \varphi_{mo}) + (4\pi \sin(m\pi d/l)/\rho S l)(m/\omega_m)\int_0^t F(\tau)\exp(-\beta_m (t-\tau)\sin(\omega_m (t-\tau))d\tau\,, \tag{9}$$

where $\beta_m = (\pi/l)^4 m^4 (rl/2\rho S), \omega_{mo}^2 = \omega_o^2(1-\alpha^2/m^2)m^4, \omega_o^2 = (EJ\pi^4)/(\rho S l^4), \omega_m = \sqrt{\omega_{mo}^2 - \beta^2{}_m}$. (10)

Under harmonic excitation $P = P_0 \sin \omega t$ the resonance solution will be written down so:

$$A_{res}^m = (2\pi P_0 \sin(m\pi d/l)/(\rho S l)\cdot(m/\sqrt{(\omega_{mo}^2-\omega^2)^2+4\beta^2{}_m\omega^2}) \tag{11}$$

Analysis of formulas (8), (10), (11) achieves that the fundamental contribution to the value of deflection y with the increasing t gives y_1, as the other components are damping fast. Pay attention that there are no evens y_m under symmetrical (*d=l/2)* excitation. Decreasing to zero ω_{10} under $\alpha \to 1$ reduces to the fact that the quality factor of system Q as a measure of internal losses sharply falls, which points to increasing of hysteresis losses. Indeed, the quality factor Q of a beam is determined practically in this case, as vibrations occur on the first free form. It is equal

$$2\pi./\Psi = Q \approx Q_1 = \omega_1/2\beta_1 = 2^{-1}(\sqrt{4E^2r^{-2}(1-\alpha^2)-1}) \tag{12}$$

Under P=0 the quality factor of system Q is high value, in view of the smallness r .

Viscous friction model with an elliptic hysteresis curve load to the following result for hysteresis loop area:

$$\mathrm{S} \approx S_1 = \pi\beta_0\,\omega\,\mathrm{A}_0^2 \tag{13}$$

$$\beta_0 = \frac{\mathrm{rJ}}{2\rho \mathrm{S}}(\pi/l)^4,\ \omega_0^2 = \frac{\mathrm{EJ}}{\rho\mathrm{S}}(\pi/l)^4, A_0 = (2\pi\sin(\pi d/l)/(\rho S l))P_0/\sqrt{\omega_0^2(1-\alpha^2)-\omega^2)^2+4\beta^2{}_0\omega^2} \tag{14}$$

Thus, the model with the equivalent viscous internal friction unambiguously points to increasing of hysteresis losses when the compressive force P is approaching to a critical force P_{kr} and reduction of the fundamental free vibration frequency into the infra-low area of frequencies of vibrations. Pay attention that everything remains correctly and in the presence of external losses.

2.2.2. Model with complex Young's modulus of elasticity

As is known [3] introduction of the complex module of elasticity lays the foundation of the presence of hysteresis losses.

Let us consider this model. Let $E = E + i\varepsilon$, ε/E is little. The equation of transverse vibrations of a rod will take form

$$E^* J\frac{\partial^4 y^*}{\partial x^4} + P\frac{\partial^2 y^*}{\partial x^2} + \rho S\frac{\partial^2 y^*}{\partial t^2} = l^{-1}F^*(t)\delta(x-d) \tag{15}$$

Let’s present y as

$$y^* = \sum_{m=1}^{\infty} y_m^*(t)\sin(m\pi d/l) = y + iq \tag{16}$$

Then

$$\ddot{y}_m^* + \Omega_m^2 y_m^* = m(2\pi F^*(t)/\rho Sl)\sin(m\pi d/l) \tag{17}$$

Here

$$\Omega_m^2 = (\pi/l)^4(\rho S)^{-1}[EJ - P(m\pi/l)^{-2} + i\varepsilon J] = m^4\omega_0^2\sqrt{(1-\alpha/m^2)^2 + (\varepsilon/E)^2}\exp(iarctg(\varepsilon/(E(1-\alpha/m^2))$$

$$\omega_m = \mathrm{Re}\,\Omega_m, \quad \beta_m = \mathrm{Im}\,\Omega_m \tag{18}$$

The quality factor of a bar on each vibration mode Q_m will be spotted by expression

$$Q_m = 0.5ctg(0.5arctg(\varepsilon/(E(1-\alpha/m^2)))\,. \tag{19}$$

Let's note as in this model with increase of time the basic contribution to y will give y_1. Therefore beam's quality factor will be evaluated under the formula

$$Q \approx Q_1 = 0.5ctg(0.5arctg(\varepsilon/(E(1-\alpha))) \tag{20}$$

Let's mark, that at $\alpha = 0$ beam's quality factor Q will be mayor quantity because of smallness of quantity ε/E. The bar's quality factor diminishes $Q \to 0.5$ at $\alpha \to 1$. In indicates increase of hysteresis curve at aspiration of compressive force to critical force.

3. Inference

The analysis of introduced results gives an explanation of increase of internal friction at approach of compressive force to critical force. It gives in sharp diminution of a natural frequency of oscillations of the modules qvasi-zero stiffness.

4. References

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